Homework #1

Due: Thursday, 10/1/09

1. Assume that the four bases A, C, T, and G occur with equal likelihood in a DNA sequence of nine monomers.

   (a) What is the probability of finding the sequence AAATCGAGT through random chance?

   (b) What is the probability of finding the sequence AAAAAAAAA through random chance?

   (c) What is the probability of finding any sequence that has four A's, two T's, two G's, and one C, such as that in (a)?

2. Suppose you roll a die three times.

   (a) What is the probability of getting a total of two **5**'s from all three rolls of the dice?

   (b) What is the probability of getting a total of *at least* two **5**'s from all three rolls of the die?

3. Consider the probability distribution $p(x) = ax^n$, $0 \leq x \leq 1$, for a positive integer n.

   (a) Derive an expression for the constant a, to normalize $p(x)$.

   (b) Compute the average $<x>$ as a function of n.

   (c) Compute $\sigma^2 = <x^2> - <x>^2$ as a function of n.

4. Your statistical mechanics class has twenty-five students. What is the probability that at least two classmates have the same birthday?

5. Suppose that you have n dice, each a different color, all unbiased and six-sided.

   (a) If you roll them all at once, how many distinguishable outcomes are there?

   (b) Given two distinguishable dice, what is the most probable sum of their face values on a given throw of the pair? (That is, which sum between two and twelve has the greatest number of different ways of occurring?)

   (c) What is the probability of the most probable sum?

6. If you flip an unbiased green coin and an unbiased red coin five times each, what is the probability of getting four red heads and two green tails?

7. What is the average value of x, given a distribution function $q(x) = cx$, where x ranges from zero to one, and $q(x)$ is normalized?

8. In forensic science, DNA fragments found at the scene of a crime can be compared with DNA fragments from a suspected criminal to determine that the probability that a match occurs by chance. Suppose that DNA fragment A is found in 1% of the population, fragment B is found in 4% of the population, and fragment C is found in 2.5% of the population. If the three fragments contain independent information, what is the probability that a suspect's DNA will match all three of these fragment characteristics by chance?

9. Given a flat distribution, from $x = -a$ to $x = a$, with probability distribution $p(x) = 1/(2a)$:

   (a) Compute $\langle x \rangle$.

   (b) Compute $\langle x^2 \rangle$.

   (c) Compute $\langle x^3 \rangle$.

   (d) Compute $\langle x^4 \rangle$.

10. A biological membrane contains N ion-channel proteins. The fraction of time that any one protein is open to allow ions to flow through is q. Express the probability $P(m,N)$ that m of the channels will be open at any given time.

11. Find the value $n = n^*$ that causes the function

$$\mathrm{p}(n) = \frac{N!}{n!\,(N-n)!}\, p^n (1-p)^{(N-n)}$$

to be at a maximum (for simplicity let $p = ½$). Use Stirling's approximation, $x! \approx \left(\frac{x}{e}\right)^x$. Note that it is easier to find the value of n that maximizes $\ln \mathrm{p}(n)$ than the value that maximizes $\mathrm{p}(n)$. The value of n^* will be the same.

12. Flip a coin $4N$ times. The most probable number of heads is $2N$, and its probability is $p(2N)$. If the probability of observing N heads is $p(N)$, show that the ratio $p(N)/p(2N)$ diminishes as N increases.

13. For the two 10-particle two-state systems described in lecture, suppose the total energy to be shared between the two objects is $E = E_A + E_B = 4$. What is the distribution of energies that gives the highest multiplicity?