

**Statistical Mechanics Home Work 1.**  
**Due Friday May 15 before we start the class.**

Problem 1

1a. (50 points) You know that the thermodynamic internal energy  $U$  is given by

$$U = \sum_{\alpha=0}^{\infty} E_{\alpha} P_{\alpha} \quad (1)$$

and that

$$c_v = \left( \frac{\partial U}{\partial t} \right)_{N,V} \quad (2)$$

Show that if you use the Eq. (1) you can rewrite Eq. 2 in the form

$$c_v = \frac{\langle E^2 \rangle - U^2}{k T^2}$$

where

$\langle E^2 \rangle$  is the average value of the square of the energy

(1 b) (20 points) Is  $\langle E^2 \rangle$  smaller or larger than  $U^2$ ? Explain your reasoning.

(1 c) (50 points) Start from

$$dS = T \delta B = T \sum_{\alpha=0}^{\infty} E_{\alpha} dP_{\alpha} \quad (4)$$

( $\delta B$  is the heat exchanged with the surrounding) and use the thermodynamic relationship

$$c_v = T \left( \frac{\partial S}{\partial T} \right)_{N,V} \quad (5)$$

to derive an expression for  $c_v$ . Show that you get the same result as in 1a.

**Problem 2.** (50 points) Solve problem 3.1 (chapter 3, problem 1). If you do not use Mathematica explain how you would do the calculation (write down in detail what a programmer needs to do to perform the calculation for you)

**Problem 3.** (20 points) Do the vibrational and the rotational energies of the molecules in a gas contribute to the pressure? In other words, if you have two containers one contains  $N$  molecules A at temperature  $T$  and the other the  $N$  molecules B at temperature  $T$ . The vibrational frequency of A is twice as large as the vibrational frequency of B. All other molecular parameters (mass, shape) are the same. By how much the pressure in one container differs from the pressure in the other container.

Problem 4. (50 points) Solve Exercise 3.7